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Gravitational Interaction Between Moving Objects in Terms of Spatial Gravitational Fields

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Analyzing two simple experimental situations we show that from Newton's law of gravitation and Special Relativity it follows that the motion of particle in an external gravitational field can be described in terms of effective spatial fields which satisfy Maxwell-like system of equations and propagate with the speed of light. The description is adequate in a linear approximation in gravitational field and in a first-order in v^2/c^2.

KEY WORDS: Post-Newtonian approximation; classical field theory; Newtonian gravity; Special Relativity; linearized gravitational field.

1. INTRODUCTION

In this paper we discuss how the gravitational interaction for an object with nonzero velocities can be described in terms of effective spatial fields. Namely, we will show that the force acting on a particle that moves in an external gravitational field is given by the expression

$$\vec{F} = m\vec{g} + m\vec{v} \times \vec{\mathcal{B}}, \tag{1}$$

where $\vec{g}$ is the gravitational field accounting for Newton's gravitational law for a particle at rest (see below (4)) and the effective field $\mathcal{B}$, appearing due to the Special Relativity effects like the magnetic field in electrodynamics, satisfies the relation (below we assume that $\vec{g}$-field is time independent)

$$\text{curl}\, \vec{\mathcal{B}} = -\eta\vec{\phi}, \quad \text{i.e.} \quad \oint \vec{\mathcal{B}}\, d\vec{l} = -\eta \int \vec{\phi}\, d\vec{A} \tag{2}$$

In (2) $\vec{\phi}$ is the flow of the unit of mass per unit of time and *dl* and *dA* stand for the line and area elements.

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We will show that

$$\eta = \frac{4\pi G_{\mathrm{N}}}{c^2}, \tag{3}$$

where $G_{\mathrm{N}} \approx 6.66 \times 10^{-11}\ \mathrm{Nm}^2\ \mathrm{kg}^{-2}$ is a Newton's constant and c is the speed of light. The small value of $4\pi G_{\mathrm{N}}/c^2$ explains why, in contrast to their electromagnetic counterparts, "gravimagnetic" effects caused by $\mathcal{B}$ are weak for moderate values of masses and velocities.

We consider the case of a weak gravitational field and neglect higher powers of v^2/c^2. In terms of General Relativity we would say that the curvature of space time nearly vanishes, so that Special Relativity can be applied with accuracy $1 - g_{00} \simeq 1$, g_{00} being the 00 component of the metric tensor $g_{\mu\nu}$. When $g_{00} \ll 1$ the curvature is almost zero, but nearly vanishing deviation from the flatness of space–time still leads to noticeable acceleration, described with accuracy v^2/c^2 by Newton's gravitational law (Dirac, 1976; Landau and Lifshitz, 1962).

In Section 2 we consider two *gedanken* experiments with the point particle and mass flow. We will evaluate η comparing the result of the first experiment with the expression (1) and then show that the results for the Experiment 2 are described by (1) with the value of η obtained from the analysis of Experiment 1. We derive equations for the fields $\vec{g}$ and $\vec{\mathcal{B}}$ that are similar to Maxwell's equations for the electromagnetic field.

In Section 3 we summarize our results and discuss the limitations of the suggested approach.

The concept of spatial gravitational forces modelled after the electromagnetic Lorentz force has a long history and many names associated with it (Bel, 1959; Bonnor, 1995; Braginsky *et al.*, 1977; Cataneo, 1958; Damour *et al.*, 1991; Dunsby *et al.*, 1997; Holzmuller, 1870; Jantzen *et al.*, 1990; Maartens *et al.*, 1997; Mashoon *et al.*, 1997; Tisserand, 1872; Zel'manov, 1956). In this paper we consider spatial gravitational fields in the most elementary way and show that even in such a simplified scheme gravitational phenomena can be analyzed with the accuracy $o(v^2/c^2)$ without invoking equations of General Relativity.

2. EXPERIMENTS WITH POINT PARTICLE AND MASS FLOW AND THE FIELD EQUATIONS

2.1. Description of Experiments

In this section we consider two *gedanken* experiments: 1) point particle moving between two infinite pipes that carry a mass flow, and 2) one pipe, moving toward the particle. We analyze these two experiments using only Special Relativity and Newton's gravitational law

$$\vec{F}_{12} = G_{\mathrm{N}} \frac{m_1 m_2}{r_{12}^3} \vec{r}_{12} \tag{4}$$

The set up for Experiment 1 is two infinite, straight, massless pipes on a plane and a point particle between them. Each of the pipes is parallel to the y axis, crossing the x axis at $x = \pm b$, and each carries a fuild that flows with velocity $v_{\rm f}$ relative to the pipe. The fluid in the left pipe moves in the positive y direction, the fluid in the right pipe moves in the negative y direction. The linear density (we neglect the pipe cross-section) of fluid at rest is σ. The particle with mass m moves along y axis with the velocity $v_{\rm p} = v_{\rm f}$.

Let us calculate the net force F acting on the particle in the frames of reference where particle is (momentarily) at rest—reference frame comoving with the particle. In this frame of reference the particle lies between two continuous mass flows with the linear densities σ and $\sigma\gamma_V$ where γ_V is a Lorentz factor accounting the relativistic length contraction

$$\gamma_V = \left(1 - \frac{V^2}{c^2}\right)^{-1/2} \tag{5}$$

In (5), $V = -\frac{2v_{\rm f}}{1+v_{\rm f}^2/c^2}$ is the velocity of a fluid from a right pipe in this frame of reference. The forces exerted from the pipes have only an x components and the straightforward calculation leads to the following expression for the force acting on a particle in the frame where particle is at rest:

$$F = \frac{2G_{\rm N}m\sigma}{b}(\gamma_V - 1) \approx \frac{4G_{\rm N}m\sigma}{b}\frac{v_{\rm f}^2}{c^2} \tag{6}$$

To obtain (6) we use expression (4), e.g. the force exerted from the left flow (second term of (6)) is

$$F_{\rm L} = -\int_{-\infty}^{+\infty} dy \frac{G_{\rm N}m\sigma b}{(b^2+y^2)^{3/2}} = -2G_{\rm N}\frac{m\sigma}{b} \tag{7}$$

The magnitude of the force exerted from the right flow is given by (7) using substitution $\sigma \to \sigma\gamma_V$ (since we consider infinite pipes there are no any boundary effects caused by the endpoints of fluid) and the final result is the expression (6).

When the particle is at rest relative to the flow the net force is zero, as it follows from (6). Below we will show that $\mathcal{B} \sim v_{\rm f}$ (see (10)) and that expression (6) is reproduced by the v-dependent term of the Eq. (1).

In the second experiment, the particle with coordinates (x, y, z) is at rest and the pipe oriented along y axis moves toward the particle with the velocity $\vec{v} = (-v_{\rm p}, 0, 0)$. The fluid with density ρ moves with the velocity $\vec{v}_{\rm f} = (0, -v_{\rm f}, 0)$ relative to the pipe.

We omit lengthy but straightforward calculations and report only the results for $\vec{a}$—the acceleration of the particle:

$$a_x \equiv \frac{dv_x}{dt} \approx -\frac{2G_{\rm N}\sigma x}{(x^2+z^2)}\left(1 + \frac{v_{\rm f}^2}{2c^2}\right)$$

$$a_y \equiv \frac{dv_y}{dt} \approx \frac{2G_N \sigma v_p v_f x}{c^2(x^2+z^2)}$$
$$a_z \equiv \frac{dv_z}{dt} \approx -\frac{2G_N \sigma z}{(x^2+z^2)}\left(1+\frac{v_p^2}{2c^2}+\frac{v_f^2}{2c^2}\right), \tag{8}$$

where x is γ_{v_p} times the distance from the pipe to the particle in the reference frame where particle is at rest. Expressions (8) are obtained by integration similar to that (7) which is based on (4), taking into account the length contraction of a small element of fluid with mass ρdy and neglecting higher orders of v^2/c^2.

2.2. Evaluation of η From the Results of Experiment 1

To obtain the value of η, appearing in the relation (2) it is necessary to consider the problem in a reference frame where particle has a nonzero velocity. The simplest solution is provided by the original frame of reference described previously when particle and the fluid from a left pipe move along y axis with velocities v_p and v_f correspondingly, $v_p = v_f$, and the fluid from the right pipe moves with the velocity $-v_f$.

From the symmetry arguments it follows that the net field $\vec{g}$ is zero (particle is between two sources with the same linear densities $\sigma\gamma$), so only the second term of (1) contributes. According to (1) the force acting on the particle is directed along x axis and its magnitude is $mv_p\mathcal{B}$. This expression already assumes that $\vec{\mathcal{B}}\vec{v}_f = \vec{\mathcal{B}}\vec{v}_p = 0$, i.e. $\vec{\mathcal{B}}$ is perpendicular to the pipes just as the magnetic field given by Ampere's circuital law is perpendicular to the current (Landau and Lifshitz, 1962). In Experiment 1 we have $\partial\vec{g}/\partial t = 0$ (pipes are at rest), so we can use (2) to calculate the value of $\vec{\mathcal{B}}$. Particle is at rest relative to the fluid in the left pipe, so it "feels" field $\mathcal{B}$ generated only by the pipe from the right. Using the integral relation (2) and for the relative velocity $V = -\frac{2v_f}{1+v_f^2/c^2}$ we obtain

$$2\pi b\mathcal{B} = -\eta\sigma\gamma V = \eta\sigma\frac{2v_f}{1+\frac{v_f^2}{c^2}} \approx 2\eta\sigma v_f, \tag{9}$$

i.e.

$$\mathcal{B} \approx \eta\sigma v_f/\pi b \tag{10}$$

and the force acting on a moving particle in the original reference frame will be (taking into account $v_p = v_f$):

$$\mathcal{F} = mv_p\mathcal{B} = \frac{\eta m\sigma v_f^2}{\pi b} \tag{11}$$

In order to establish the value of η we have to compare F and $\mathcal{F}$—force calculated in a two reference frames. Reference frames move relative to each other along

y axis with velocity V and the force is directed along x axis. Thus we need Lorentz transformations in its vectorial form:

$$t' = \gamma\left(t - \frac{(\vec{V}\vec{r})}{c^2}\right); \qquad \vec{r}' = \gamma\left(\gamma^{-1}\vec{r} - \vec{V}t + (1-\gamma^{-1})\frac{(\vec{V}\vec{r})\vec{V}}{V^2}\right) \tag{12}$$

From (12) and the expression for the force

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\gamma\vec{v})}{dt} = m\gamma\vec{a} + m\gamma^3\frac{(\vec{v}\vec{a})\vec{v}}{c^2}, \tag{13}$$

it follows that up to order v^4/c^4 (let us remind that F itself is of order v^2/c^2) we have $F = \mathcal{F}$. Therefore, equating expressions for F and $\mathcal{F}$ we obtain

$$\eta = \frac{4\pi G_\mathrm{N}}{c^2}\left(1 + o\left(\frac{v^2}{c^2}\right)\right) \tag{14}$$

2.3. Description of Experiment 2 in Terms of $\vec{g}$ and $\vec{\mathcal{B}}$

After the value of η is established we demonstrate that the results for Experiment 2 are described by (1). To do so, instead of straightforward but lengthy arguments we assume that when $\partial\vec{g}/\partial t \neq 0$ (in the reference frame used to describe Experiment 2, the pipe has nonzero velocity, so $\vec{g}$ is now time dependent) expression (2) is modified as

$$\mathrm{curl}\,\vec{\mathcal{B}} = \frac{1}{c^2}\frac{\partial\vec{g}}{\partial t} - \eta\vec{\phi}, \quad \text{i.e.} \quad \oint \vec{\mathcal{B}}\,d\vec{l} = \int\left(\frac{1}{c^2}\frac{\partial\vec{g}}{\partial t} - \eta\vec{\phi}\right)\,d\vec{A} \tag{15}$$

We will verify that (1) and (15) lead to the same expressions for the force as did calculation using only Newton's law and Special Relativity.

The components of $\vec{\mathcal{B}}$ can be calculated from (15). For $\vec{g}$ we use $\vec{g} = \vec{a}$ with $\vec{a}$ given by (8) where $d\vec{g}/dt = (d\vec{g}/dx)\cdot(dx/dt)$, and assume that fields vanish at infinity. The choice $\vec{g} = \vec{a}$ is justified since in the original frame of reference particle is at rest and its (instantaneous) acceleration is defined by a "$\vec{g}$ term" of (1). Using $\eta = 4\pi G_\mathrm{N}/c^2$, in a leading approximation in v^2/c^2 we obtain

$$\begin{aligned}
\mathcal{B}_x &\approx \frac{2G_\mathrm{N}\sigma v_\mathrm{f} z}{c^2(x^2+z^2)}\\
\mathcal{B}_y &\approx -\frac{2G_\mathrm{N}\sigma v_\mathrm{p} z}{c^2(x^2+z^2)}\\
\mathcal{B}_z &\approx -\frac{2G_\mathrm{N}\sigma v_\mathrm{f} x}{c^2(x^2+z^2)}
\end{aligned} \tag{16}$$

Now we are in a position to check that Experiment 2 can be described by $\vec{F} = m\vec{g} + m\vec{v}\times\vec{\mathcal{B}}$.

When the particle is at rest in the reference frame used to describe Experiment 2, $\vec{F} = m\vec{g}$, which is trivially consistent with the Eq. (8), since we have defined $\vec{g} = \vec{a}$.

To account the effect caused by the "$\mathcal{B}$-term" of (1), we consider the case when particle in the original frame of reference moves with the nonzero velocity $\vec{u} = (u, 0, 0)$. The y component of (1) is $F_y = mg_y - m\mathcal{B}_z u$. Direct substitution for $\mathcal{B}_z$ (and $g_y = a_y$) results in

$$F_y = m\frac{2G_{\mathrm{N}}\sigma v_{\mathrm{p}} v_{\mathrm{f}} x}{c^2(x^2+z^2)} + mu\frac{2G_{\mathrm{N}}\sigma v_{\mathrm{f}} x}{c^2(x^2+z^2)} \tag{17}$$

To compare (17) with the expression calculated in the framework of Newtonian approximation, $F_y = ma_y$, we need the value of a_y. From (8), acceleration in case of $\vec{u} = 0$, it follows that when a particle has nonzero $\vec{u} = (u, 0, 0)$, the value of a_y can be obtained by substitution of $v_{\mathrm{p}} + u$ for v_{p} in (8): $v_{\mathrm{p}} \to v_{\mathrm{p}} + u$ that leads to

$$F_y = ma_y(v_{\mathrm{p}} \to v_{\mathrm{p}} + u) = m\frac{2G_{\mathrm{N}}\sigma (v_{\mathrm{p}}+u) v_{\mathrm{f}} x}{c^2(x^2+z^2)} \tag{18}$$

As it is clear, (17) and (18) agree.

Next we consider the z component. Using (16) for B_y we obtain

$$F_z = mg_z + muB_y = -m\frac{2G_{\mathrm{N}}\sigma z}{x^2+z^2}\left(1 + \frac{v_{\mathrm{p}}^2}{2c^2} + \frac{v_{\mathrm{f}}^2}{2c^2}\right) - m\frac{2G_{\mathrm{N}}\sigma v_{\mathrm{p}} z}{c^2(x^2+z^2)}u \tag{19}$$

Since the particle moves with the velocity $\vec{u} = (u, 0, 0)$ we have $a_z = F_z/\gamma_u m$ (see (13)):

$$a_z \approx -\frac{2G_{\mathrm{N}}\sigma z}{x^2+z^2}\left(1 + \frac{v_{\mathrm{p}}^2}{2c^2} + \frac{v_{\mathrm{f}}^2}{2c^2} - \frac{u^2}{2c^2}\right) - \frac{2G_{\mathrm{N}}\sigma v_{\mathrm{p}} z}{c^2(x^2+z^2)}u, \tag{20}$$

where $\gamma_u^{-1} \equiv \sqrt{1 - u^2/c^2} \approx 1 - u^2/2c^2$.

Now we have to compare this expression with the one for a_z from (8), calculated from Newton's law and Special Relativity—acceleration of a particle in the reference frame comoving with the particle. We replace $v_{\mathrm{p}} \to v_{\mathrm{p}} + u$ in the expression (8) for a_z to obtain

$$a_z(v_{\mathrm{p}} + u) = -\frac{2G_{\mathrm{N}}\sigma z}{x^2+z^2}\left(1 + \frac{v_{\mathrm{p}}^2}{2c^2} + \frac{v_{\mathrm{f}}^2}{2c^2} + \frac{v_{\mathrm{p}} u}{c^2} + \frac{u^2}{2c^2}\right) \tag{21}$$

Expression (21) gives the acceleration of the particle in a reference frame moving along the x axis with velocity $v_{\mathrm{p}} + u$ relative to the pipe. On the other hand, expression (20) corresponds to the acceleration in the reference frame moving along the x axis with velocity v_{p} relative to the pipe. To compare (20) and (21), we use (12) to transform acceleration from the reference frame used in (21) to that used in (20): $a_z \to a_z' \gamma_u^{-2}$. This transformation introduces the term $-u^2/c^2$ so that the two expressions for acceleration now agree.

A similar analysis for the case when the velocity of the particle is along the y axis, $\vec{u} = (0, u, 0)$ again confirms that the expression $\vec{F} = m\vec{g} + m\vec{v} \times \vec{\mathcal{B}}$ can be used to describe the motion of a particle in a gravitational field.

2.4. Equations for $\vec{g}$ and $\vec{\mathcal{B}}$

Besides Eq. (15) that was postulated (and subsequently verified to describe Experiment 2 in a self-consistent way) it is possible work out two more relations for the fields $\vec{\mathcal{B}}$ and $\vec{g}$.

First of all, from the expressions (16) it follows that $\partial\mathcal{B}_x/\partial x + \partial\mathcal{B}_y/\partial y + \partial\mathcal{B}_z/\partial z = 0$, i.e.

$$\operatorname{div} \vec{\mathcal{B}} = 0 \tag{22}$$

Next we compare curl $\vec{g}$ and $\partial\vec{\mathcal{B}}/\partial t$. For the y component we obtain

$$\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial t}\left(-\frac{2G_{\mathrm{N}}\rho v_{\mathrm{p}} z}{c^2(x^2+z^2)}\right) = \frac{2G_{\mathrm{N}}\rho v_{\mathrm{p}}^2 2zx}{c^2(x^2+z^2)^2} \tag{23}$$

Straightforward calculation of the y component of curl $\vec{g}$ (as before, we take $\vec{g} = \vec{a}$, for $\vec{a}$ see (8)) results in

$$\begin{aligned}\frac{\partial g_x}{\partial z} - \frac{\partial g_z}{\partial x} &= \frac{\partial}{\partial z}\left(-\frac{2G_{\mathrm{N}}\rho x}{(x^2+z^2)}\left(1+\frac{v_{\mathrm{f}}^2}{2c^2}\right)\right) - \frac{\partial}{\partial x}\left(-\frac{2G_{\mathrm{N}}\rho z}{(x^2+z^2)}\right)\\ &\approx -\frac{2G_{\mathrm{N}}\rho 2zx}{(x^2+z^2)^2}\frac{v_{\mathrm{p}}^2}{c^2},\end{aligned} \tag{24}$$

i.e. $(\operatorname{curl} \vec{g})_y = -\partial\mathcal{B}_y/\partial t$.

Consideration of the x and z components show that the relation

$$\operatorname{curl} \vec{g} = -\frac{\partial\vec{\mathcal{B}}}{\partial t} \tag{25}$$

is valid. Also, from the definition of $\vec{g}$ we have $\operatorname{div} \vec{g} = 4\pi G_{\mathrm{N}}\rho$ where ρ is a regular three-dimensional density.

Summarizing, the equations for $\vec{g}$ and $\vec{\mathcal{B}}$ are as follows:

$$\begin{aligned}&\operatorname{div} \vec{g} = 4\pi G_{\mathrm{N}}\rho, \qquad \operatorname{curl} \vec{g} = -\partial\vec{\mathcal{B}}/\partial t\\ &\operatorname{div} \vec{\mathcal{B}} = 0, \qquad \operatorname{curl} \vec{\mathcal{B}} = \frac{1}{c^2}\frac{\partial\vec{g}}{\partial t} - \frac{4\pi G_{\mathrm{N}}}{c^2}\vec{j}\end{aligned} \tag{26}$$

where ρ is the mass density and $\vec{j}$ is the mass density flow. In case of Experiments 1 and 2 $\vec{j} = \vec{\phi} = \rho\vec{v}_{\mathrm{f}}$.

It is now straightforward to obtain the wave equations for the case $\rho = 0$, $\vec{j} = 0$

$$\frac{1}{c^2}\frac{\partial^2 \vec{g}}{\partial t^2} = \nabla^2 \vec{g}, \qquad \frac{1}{c^2}\frac{\partial^2 \vec{\mathcal{B}}}{\partial t^2} = \nabla^2 \vec{\mathcal{B}}, \tag{27}$$

i.e. free waves propagate with speed of light.

3. DISCUSSION

We have demonstrated that the gravitational force acting on a point particle with mass m and velocity $\vec{v}$ is given by the expression

$$\vec{F} = m\vec{g} + m\vec{v} \times \vec{\mathcal{B}} \tag{28}$$

with $\vec{g}$ and $\vec{\mathcal{B}}$ satisfying the system of equations similar to the Maxwell equations.

The approximation we used is that gravitational field is weak enough so that space-time is approximately euclidean and the velocities are small enough so that higher powers of v^2/c^2 are negligible. In the framework of this approximation the force obtained from Newton's law (4) and the Special Relativity is described by (28), i.e. motion of particle is given by an expression similar to the Lorentz force for a charged particle in an external electromagnetic field. The similarity is caused by neglecting the effects of self-interaction for gravitational field, corresponding to a nonlinearity of Einstein's equations. In case of classical electromagnetism the linear approximation to field equations is well justified in a sense that phenomena with characteristic action substantially exceeding $\hbar$, $\hbar$ being the Planck's constant, are described by Maxwell's and Lorentz's equations (Landau and Lifshitz, 1962) and in electromagnetic phenomena quantum effects manifest themselves earlier than effects caused by a nonlinear corrections to Maxwell's equations. Intuitively it becomes clear when one compares electron's Compton wave length $r_{\mathrm{q}} = \hbar/mc$ and its classical electromagnetic radius $r_{\mathrm{e}} = e^2/mc^2$: from the value of the ratio $r_{\mathrm{q}}/r_{\mathrm{e}} = \hbar c/e^2 \approx 137$ it follows that the quantum effects, namely the pair production occurs at a distance that is 137 times more than the distance at which the classical field singularities become relevant and when it becomes necessary to modify classical theory, e.g. to introduce nonlinear terms in field equations.

Theory of gravity provides us with an opposite feature—"classical radius" $r_{\mathrm{g}} = 2G_{\mathrm{N}}m/c^2$ appearing in the Schwarzschild's metric (Dirac, 1976; Landau and Lifshitz, 1962) greatly exceeds Compton wavelength—$r_{\mathrm{g}}/r_{\mathrm{q}} = 2m^2/M_{\mathrm{Pl}}^2$, where the Planck mass $M_{\mathrm{Pl}} \approx 10^{-5}$ g. Therefore in describing the motion of bodies with $m \gg M_{\mathrm{Pl}}$ it is vital to consider the exact classical equations of motion (Einstein's nonlinear equations)—quantum effects are negligible at this scale. The self-interaction plays a decisive role in describing basic phenomena of light deflection or precession of perihelion of planetary motion (Dirac, 1976; Landau and Lifshitz, 1962). This features of a motion in a *static* gravitational field cannot be described by (28)—the "$\vec{\mathcal{B}}$ term" is absent for a static source. Therefore, fields $\vec{g}$ and $\vec{\mathcal{B}}$, describing gravitational interaction of a moving object in a linear approximation, can be treated only as an effective fields and the limitation of our approach manifests itself in degrees of freedom: 6 components of $\vec{g}$ and $\vec{\mathcal{B}}$ of course are not enough to describe the degrees of freedom of a gravitational field.

Despite that the relevancy of the linear approximation is questionable, approximation (28) can be still useful for describing interaction of moving bodies: from General Relativity it follows that the exact expression for the force exerted on point particle moving in an external stationary field is given by expression similar to (28) (Landau and Lifshitz, 1962):

$$\vec{F} = -mc^2 \vec{\nabla} \ln\sqrt{-g_{00}} + mc\sqrt{-g_{00}}\vec{v} \times \text{curl}\, \vec{\mathcal{G}}, \tag{29}$$

where $\mathcal{G}_\alpha \equiv -g_{\alpha 0}/g_{00},\ \alpha$ stands for a spatial part of metric and $g_{\mu\nu}$ is a metric tensor. When $g_{00} = -1 - 2\Phi/c^2$, where Φ is a scalar potential, $\Phi/c^2 \ll 1$, the first terms of the r.h.s. of (29) is the same as the first term of the r.h.s. of (28). To reproduce the second term of (28) which includes the field $\vec{\mathcal{B}}$, that is to express $\vec{\mathcal{B}}$ in terms of $g_{\mu\nu}$ it would be necessary to solve Einstein's equations. At the moment, we know of no solutions for the Einstein's equations for Experiments 1 and 2, but based on our phenomenological consideration we believe that the equation similar to (28) can be derived from the equations of General Relativity.

Let us note that though fields $\vec{g}$ and $\vec{\mathcal{B}}$ satisfy wave equations (27), they do not transform as an antisymmetric tensor of rank 2, i.e. they do not transform as the electromagnetic field $F_{\mu\nu} \sim (\vec{E}, \vec{H})$. If one attempts to postulate that the *exact* expression for the force acting on a test particle is given by (28) or (29) then it turns out that in order to maintain expression (28) fields $\vec{g}$ and $\vec{\mathcal{B}}$ transform like nontensor quantities (Dirac, 1976). This is the price one has to pay when attempting to describe gravitational interaction in terms of six degrees of freedom. The nontensor feature of transformation is most transparent from (29): identifying $\vec{g}$ with the first term of r.h.s. of (29) it follows that at $x^\mu \to x^\mu + \xi^\mu(x)$ in the expression for the transformed $\vec{g}$ there arises extra term

$$\delta g_i(x) = \frac{\partial}{\partial x^i}\left(\frac{g^{\mu 0}}{g^{00}}\frac{\partial \xi^\mu}{\partial x^\mu}\right) \ln\sqrt{-g_{00}}, \tag{30}$$

which cannot be compensated by the transformation of a "$\vec{\mathcal{B}}$-term" of (29). Therefore Eq. (28) cannot hold in any reference frame, for *any* velocities. In Einstein's equations extra terms similar to (30) are compensated by coordinate transformations of General Relativity and as a result, equations of gravitational field and the requirement of general covariance form a self consistent mathematical scheme (Dirac, 1976; Landau and Lifshitz, 1962).

In approximation used in this paper (linearized equations and lowest order in v^2/c^2) fields $\vec{g}$ and $\vec{\mathcal{B}}$ transform as $\vec{E}$ and $\vec{H}$. This statement is true only in lowest order in v^2/c^2. Straightforward calculation shows that (8) and (16) transform exactly as $\vec{E}$ and $\vec{H}$ transform in the lowest order in v^2/c^2. Since the fields $\vec{g}$ and $\vec{\mathcal{B}}$ are defined in the framework of this approximation, the description based on (28) and (26) is self-consistent in the linear approximation and up to higher orders in v^2/c^2.

As we already have mentioned, $\vec{g}$ and $\vec{\mathcal{B}}$ are effective fields, even from the point of view of classical theory. Nevertheless, Eq. (28) can be applied to a rather wide class of phenomena in problem of describing the motion in external gravitational field after the fields $\vec{g}$ and $\vec{\mathcal{B}}$ are known. The advantage of using (28) and (26) is in their simplicity in comparison with the problem of solving equations of General Relativity.

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